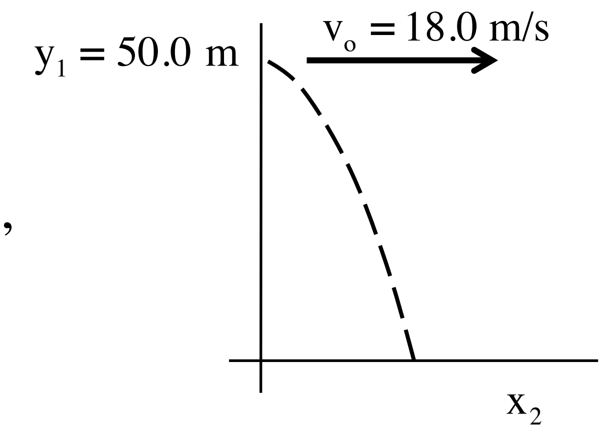
Problem 4.23



a.) The stone's initial coordinate is:

$$\vec{r}_1 = 0\hat{i} + (50.0\text{ m})\hat{j} \qquad \text{or} \qquad \vec{r}_1 = (50.0\text{ m})\measuredangle 90^\circ,$$

depending upon the notation you use.

b.) The stone's initial velocity is:

$$\vec{v}_1 = (18.0\text{ m/s})\hat{i} + 0\hat{j} \qquad \text{or} \qquad \vec{v}_1 = (18.0\text{ m/s})\measuredangle 0^\circ.$$

c.) What needs to be observed for the vertical motion?

The acceleration is constant, so kinematics with a constant downward acc "g" can be used.

The math

$$\cancel{y_2}^{\,0} = y_1 + \cancel{v_{y,1}}^{\,0}\Delta t + \frac{1}{2}(-g)(\Delta t)^2$$

$$\Rightarrow \quad 0 = (50.0\text{ m}) + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)(\Delta t)^2$$

d.) What needs to be observed for the horizontal?

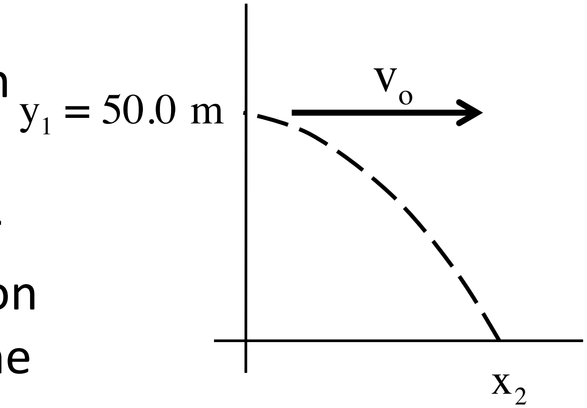
The acceleration is constant, so kinematics with a constant acceleration of "zero" can be used.

The math

$$x_2 = \cancel{x_1}^{\,0} + v_{1,x}\Delta t + \frac{1}{2}\cancel{a_x}^{\,0}(\Delta t)^2$$

$$\Rightarrow \quad x_2 = (18.0\text{ m/s})\Delta t$$

e.) Symbolic relationship for velocity of stone as function of time:



The velocity component in the x-direction will never change as there are no forces acting in the x-direction (we are ignoring friction). In the y-direction, the time dependent velocity will be:

$$v_{2,y} = \cancelto{0}{v_{1,y}} + a_y \Delta t$$

$$\Rightarrow \quad v_{2,y} = (-g)\Delta t$$

Putting everything together, noting that the speed in the x-direction is always 18.0 m/s, we get:

$$\vec{v}(t) = (18.0 \text{ m/s})\hat{i} - (g\Delta t)\hat{j}.$$

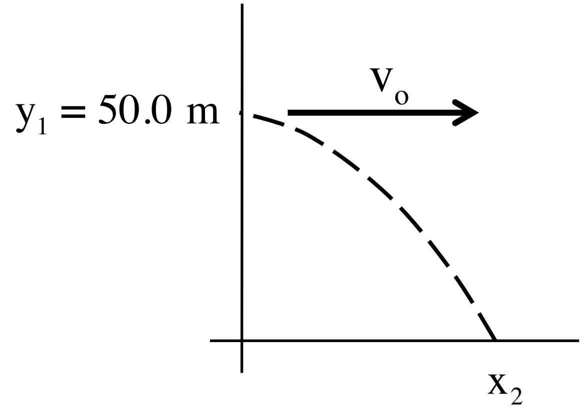
f.) Symbolic relationship for position of stone as function of time:

From Parts "c" and "d:"

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$= \left[(18.0 \text{ m/s})\Delta t\right]\hat{i} + \left[(50.0 \text{ m}) + \frac{1}{2}(-g)(\Delta t)^2\right]\hat{j}$$

g.) Time of flight?



This has to do solely with what is happening in the y-direction. As such, we can write:

$$\cancel{y_2}^{0} = y_1 + \cancel{v_{y,1}}^{0}\Delta t + \frac{1}{2}(-g)(\Delta t)^2$$

$$\Rightarrow \quad 0 = (50.0\text{ m}) + \frac{1}{2}(-9.80\text{ m/s}^2)(\Delta t)^2$$

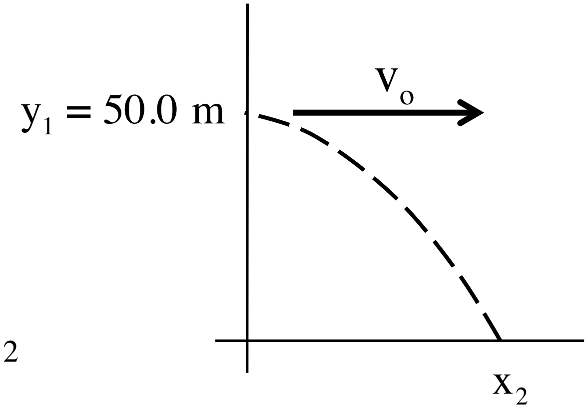
$$\Rightarrow \qquad \Delta t = 3.19\text{ s}$$

h.) "just before impact," what's the speed and angle.

This is usually done in a unit vector notation because you always know the x-component. It means, though, that you'll have to convert to polar to actually answer the question as stated. In any case, in the x-direction:

$$v_x = (18.0\text{ m/s})\hat{i}.$$

In the y-direction:



$$\left(v_{2,y}\right)^2 = \left(\cancelto{0}{v_{1,y}}\right)^2 + 2a_y\left(\cancelto{0}{y_2} - y_1\right)$$

$$\Rightarrow \quad v_{2,y} = \left[2(-g)(-h)\right]^{1/2}$$

$$\Rightarrow \quad v_{2,y} = \left[2\left(-9.80\text{ m/s}^2\right)\left(-50.0\text{ m}\right)\right]^{1/2}$$

$$= 31.3\text{ m/s}$$

(Note that if you had just identified the height "h," you may have made the mistake of thinking that $\Delta y = h$, which it isn't . . . it's $\Delta y = \left(y_2 - y_1\right) = \left(0 - h\right)$. As I pointed out earlier, it's always better to write Δ's in terms of the variables that belong to them (coordinates if it's a Δy, velocities if it's a Δv). In any case, after manually inserting the negative sign for the y-component, we can write:

$$\vec{v}(t) = \left(18.0\text{ m/s}\right)\hat{i} - \left(31.3\text{ m/s}\right)\hat{j}$$

or

$$\vec{v}(t) = \left[\left(18.0\text{ m/s}\right)^2 + \left(-31.3\text{ m/s}\right)^2\right]^{1/2} \measuredangle \tan^{-1}\left(\frac{-31.3}{18}\right)$$

$$= \left(36.1\text{ m/s}\right) \measuredangle -60^{\circ}$$